On some Applications of Laplace-Transformation to the Heat Transfer Problems. (Report 4)

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Introduction.

The fundamental studies for heat-exchangers have been performed by W. Nusselt, H. Hausen and G. Ackermann, and these energy equations have the same type differential equation on each cases, especially for the cross-flow heat-exchanger the theoretical researches has dealt with many authors.

Many types of heat-exchanger for gas turbine, such as regenerative matrix or plate-and-fin type exchangers, are considered as the special cases of recuperative ones.

Solving these fundamental energy equations, the writer uses method of Laplace-Transformation, and for the regenerative exchanger the dual or two-dimensional transformation is very effective method.

For the final numerical calculation the solution of serial form are most convenient one.

1. Fundamental heat energy equation.

In Fig.1 matrix type exchanger rotates with slow speed and working fluids, i.e., heating gas and cooling liquid, apply to both sides of rotor periodically.

Gas and air flows parallel on the same direction, say annular type, the effectiveness shows the lower value, but these flows take counter flow, say disc type, the effectiveness
indicates the higher values and used for heat exchanger of general gas turbines.\(^{(60)}\)

In the heating period the equation of exchanged heat energy between the gas and matrix is as follows;\(^{(44)}\)

\[
-\frac{\alpha_h A_h}{L} (\vartheta_h - \vartheta_r) = - \frac{M_C}{L} \frac{\partial \vartheta_r}{\partial t} = - \left( W_h C_h \frac{\partial \vartheta_h}{\partial x} + \rho_h C_f A_f \frac{\partial \vartheta_h}{\partial t} \right)
\]  

(83)

where, \(\vartheta_h, \vartheta_r\) = temperatures of gas and matrix, °C

\(\alpha_h\) = heat transmission coefficient by convection at heating period, Kcal/m² h °C

\(A_h\) = heating area, m²

\(M_c\) = weight of matrix, kg

\(C_r, C_h\) = specific heat of matrix and gas, Kcal/kg °C

\(\rho_h\) = density of gas, kg/m³

\(W_h\) = flow rate of gas, kg/h

\(A_f\) = sectional area of flowing, gas m²

\(L\) = flowing gas length, m

\(t\) = heating time, h

\(C_h = W_h C_h\) = water equivalent, Kcal/h °C

Denoting a heating period by \(\tau_{oh}\) hour, equation (83) becomes

\[
\vartheta_h - \vartheta_r = a \frac{\partial \vartheta_r}{\partial t} = - b \frac{\partial \vartheta_h}{\partial x} - c \frac{\partial \vartheta_h}{\partial t}
\]

(84)

where, \(a = M_C/\alpha_h A_h \tau_{oh}\), \(b = W_h C_h/\alpha_h A_h\) and \(c = \rho_h C_f/\alpha_h A_h \tau_{oh}\) represent non-dimensional numbers and \(x/L = x\) only.

Similarly, at the cooling period the energy equation is

\[
\vartheta_c - \vartheta_r = a' \frac{\partial \vartheta_c}{\partial t} = b' \frac{\partial \vartheta_c}{\partial x} + c' \frac{\partial \vartheta_c}{\partial t}
\]

(85)

where, \(a' = M_C/\alpha_c A_c \tau_{oc}\), \(b' = W_i C_c/\alpha_c A_c\) and \(c' = \rho_c A_f C_c/\alpha_c A_c \tau_{oc}\).

Boundary and initial conditions now considered are as follows

\(x = 0\) ; \(\vartheta_x = \vartheta_0\) gas temperature

\(x = 1\) ; \(\vartheta_x = \vartheta_a = 0\), air temperature

and,

\(t = 0\) ; \(\vartheta_r = f(x) = \vartheta_r\mid \tau_{oc}\) for heating period

\(= g(x) = \vartheta_r\mid \tau_{oh}\) for cooling period

(86)

2. Theoretical solution.

Modifying the equation (84), equation refering to \(\vartheta_r\) is

\[
\partial^2 \vartheta_r/\partial x^2 + \frac{c}{b} \partial \vartheta_r/\partial t + \frac{1}{a} \partial \vartheta_r/\partial x + \frac{a+c}{ab} \partial \vartheta_r/\partial t = 0
\]

(87)

Now we apply the Laplace-Transformation, if \(\Theta(u, \nu)\) shows two-dimensional transformed temperature of \(\vartheta_r(x, t)\), the transformed equation of (87) is\(^{(60)}\)
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\[ u \bar{\Theta}(u,v) - u \varphi, _1(u,o) - v \varphi, _2(o,v) + \varphi, _1(o,o) + \frac{1}{a} [u \bar{\Theta}(u,v) - \varphi, _2(o,v)] + \frac{a+c}{ab} [v \bar{\Theta}(u,v) - \varphi, _1(o,o)] + \frac{c}{b} \left[ v \bar{\Theta}(u,v) - v \varphi, _2(u,o) - \frac{v \varphi, _1(u,o)}{\partial t} \right] = 0 \]  \hspace{1cm} (88)

And as the transformed conditions is

\[ x = 0 ; \quad \varphi, _1(u,o) = f(u) \]

\[ t = 0 ; \quad \varphi, _2(o,v) = \varphi, _2(o,v) - \varphi, _1(v) + \frac{1}{a} - f(o) / v + \frac{1}{a} \]  \hspace{1cm} (89)

the transformed temperature \( \Theta(u,v) \) is obtained algebraically

\[ \Theta(u,v) = \frac{b}{v} \varphi, _1 + \frac{(abu + a + c + av) f(u)}{abuv + bu + (a + c) v + av} \]  \hspace{1cm} (90)

This two-dimensional transformation method equivalent to the method of writer’s dual operational method, i.e., if we takes the operators \( \dot{p} = \partial / \partial x \) and \( s = \partial / \partial t \), the equation (83) can be represented by the following transformed equation

\[ \varphi, _1(p,s) - \varphi, _1(p,s) = a \left\{ s \varphi, _1(p,s) - f(p) \right\} = -b \left\{ p \varphi, _1(p,s) - \frac{\varphi, _2}{s} \right\} \]

\[ -c \left\{ s \varphi, _2(p,s) - \varphi, _1(p,o) \right\} \]  \hspace{1cm} (91)

Transformed temperatures \( \varphi, _1(p,s) \) and \( \varphi, _1(p,s) \) are obtained algebraically as follows

\[ \varphi, _1(p,s) = \frac{b}{s} \varphi, _1 + \frac{(a + abp + acs) f(p) + c \varphi, _1(p,o)}{as + (bp + cs)(as + 1)} \]  \hspace{1cm} (92)

and,

\[ \varphi, _1(p,s) = \frac{b \varphi, _1}{s} \frac{(1 + as) + (v + acs) f(p) + c \varphi, _1(p,o)}{as + (bp + cs)(as + 1)} \]

where, \( \varphi, _1(p,o) = f(p) \) and in the eq. (91) \( u \) and \( v \) correspond to the values \( p \) and \( s \) in the eq. (92).

To solve these equations, we rewrite eq. (91) or (92)

\[ \varphi, _1(p,s) = \frac{s}{p} \frac{(1 + as) + \frac{ab f(p)}{as + (bp + cs)(as + 1)} + \frac{p}{\frac{a+c+acs}{p} \left\{ as + (bp + cs)(as + 1) \right\}}}{p + \frac{cs}{b(1 + as) + \frac{ab f(p)}{as + (bp + cs)(as + 1)}}} \]

In the above equation we find the inverse transformation on every terms.

As,
\[ e^{-cxs/b} \begin{array}{ccc} & s \to & t \end{array} \int_0^{2\sqrt{xt/ab}} \]

for \(0 < t < cx/b\)

and,

\[ \frac{d}{d\alpha} \left[ f(\alpha) \star g(\alpha) \right] = f(\alpha) \star \frac{2g'(\alpha)}{\alpha} + f(\alpha) g(0) \]

the transformation \(p \to x\) and \(s \to t\) are as follows

\[ \frac{\partial x}{\partial s} = \frac{(1+as)}{b(1+as) + cs} \]

\[ \frac{\partial p}{\partial x} = \frac{s}{s(1+as)} \frac{x}{e} \]

\[ -\left( \frac{1}{b} - \frac{c}{ab} \right)x - \frac{t}{a} \int_0^t \int_0^{2\sqrt{xc/ab}} \frac{\tau}{\alpha} d\tau \]

\[ p \left[ \frac{ab f(p)}{as+(bp+cs)(as+1)} \right] + p \left[ \frac{a+c+acs}{p(as+(bp+cs)(1+as))} \right] \]

\[ \frac{p}{s} \to x \frac{f(x)}{s + \frac{1}{a}} + \frac{a+c(1+as)}{(1+as)^2 b} \frac{as}{e} \]

\[ x \times f(x) \]

\[ \frac{t}{a} e^{\frac{t}{a}} \int_0^x \left( \frac{1}{b} - \frac{c}{ab} \right) e^{\frac{t}{a}} \int_0^{2\sqrt{xt/ab}} d\tau \]

\[ \int_0^{2\sqrt{xt/ab}} f(x-\xi) d\xi \]

where \(\star\) means convolution refer to \(x\) and \(I_0\) is the modified Bessel Function of the second kind.

In these equations, put \(c=0, \quad \partial_s=0\) and \(f(x)=1.0\)

\[ \partial_{ro}(p,s) = \frac{a+abp}{as+bp(1+as)} \quad p \to x \frac{1}{s} \left( 1 - \frac{1}{1+as} e^{-\frac{as}{b(1+as)x}} \right) \]

and,

\[ \partial_{ro}(p,s) = \frac{a}{as+bp(1+as)} \quad p \to x \frac{1}{s} \left( 1 - e^{-\frac{as}{b(1+as)x}} \right) \]

\(\partial_{ro}(p,s)\) and \(\partial_{ho}(p,s)\) correspond to \(\partial_1(p,s)\) and \(\partial_2(p,s)\) respectively in the case of recuperative cross-flow heat exchanger and this result shows the same solution as W. Nusselt's one.\(^{60}\)

Finally, the temperature of matrix is obtained as follows
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\[ \theta(x, t) = \theta \left( x \right) e^{-\frac{\left( \frac{1}{a} - \frac{c}{b} \right) x}{t}} \int_0^t \frac{1}{I_0(2\sqrt{\frac{xt}{ab}})} e^{-\frac{y}{a}} dy 
+ f(x) e^{-\frac{1}{a}} \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} x \int_0^t I_0(2\sqrt{\frac{xt}{ab}}) e^{-\frac{1}{b} - \frac{c}{b}} x \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} I_0(2\sqrt{\frac{xt}{ab}}) f(x - \xi) d\xi d\tau 
+ \frac{c}{ab} e^{-\frac{1}{a}} \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} I_0(2\sqrt{\frac{xt}{ab}}) f(x - \xi) d\xi 
\]

for \( t > \frac{c}{b} x \)

and,

\[ \theta(x, t) = f(x) \]

for \( 0 < t < \frac{c}{b} x \) \hspace{1cm} (93)

And the gas temperature is

\[ \theta_g(x, t) = \theta_t(x, t) + a \frac{\partial \theta_t(x, t)}{\partial t} = \theta \left( x \right) e^{-\frac{\left( \frac{1}{a} - \frac{c}{b} \right) x}{t}} \int_0^t \frac{1}{I_0(2\sqrt{\frac{xt}{ab}})} e^{-\frac{y}{a}} dy 
+ f(x - \xi) d\xi - \frac{c}{b} e^{-\frac{1}{a}} \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} x \int_0^t I_0(2\sqrt{\frac{xt}{ab}}) e^{-\frac{1}{b} - \frac{c}{b}} x \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} I_0(2\sqrt{\frac{xt}{ab}}) f(x - \xi) d\xi d\tau 
+ \frac{c}{ab} e^{-\frac{1}{a}} \int_0^x e^{-\frac{1}{b} - \frac{c}{b}} I_0(2\sqrt{\frac{xt}{ab}}) f(x - \xi) d\xi 
\]

for \( t > \frac{c}{b} x \)

and,

\[ \theta_g(x, t) = 0 \] \hspace{1cm} (94)

Next, conventional serial form of \( \theta_t(x, t) \) and \( \theta_g(x, t) \) are obtained by expanding term \( e^{-\frac{a}{b(1+b)}} x \) with 1/s.

\[ \frac{1}{s} e^{-\frac{a}{b(1+b)} n} = \sum_{n=0}^{\infty} \left( \frac{1}{s} \right)^n \left( \frac{x^t}{ab} \right)^n 
+ \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} (-1)^{n+m-1} \]

where,

\[ \sum n, m(x, t) = \sum_{n=1}^{\infty} \left( \frac{1}{n!} \right)^2 \left( \frac{xt}{ab} \right)^n + \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} (-1)^{n+m-1} \]
Therefore the matrix temperature of serial form is

\[
\begin{align*}
\partial_r (x, t) &= \frac{\partial^2}{\partial a^2} e^{-\frac{x}{b}} \int_0^t e^{-\frac{t-r}{a}} \left[ 1 + \sum n, m(x, \tau) \right] d\tau + \frac{1}{ab} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \\
&= e^{-\frac{t}{b} - \frac{r}{a}} \left[ 1 + \sum n, m(\xi, \tau) \right] f(x-\xi) d\xi d\tau - \frac{1}{a^2 b} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \\
&= e^{-\frac{t}{b} - \frac{r}{a}} \left[ 1 + \sum n, m(\xi, \tau) \right] \frac{c}{a b} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \\
&= e^{-\frac{t}{b} - \frac{r}{a}} \left[ 1 + \sum n, m(\xi, t) \right] f(x-\xi) d\xi d\tau, \\
&\text{where, } t > \frac{c}{b} x
\end{align*}
\]  

(97)

And the gas temperature is

\[
\begin{align*}
\partial_n (x, t) &= \partial_0 e^{-\frac{x}{b}} \left[ 1 + \sum n, m(x, t) \right] + \frac{1}{b} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \\
f(x-\xi) d\xi - \frac{1}{a b} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \left[ 1 + \sum n, m(\xi, \tau) \right] \\
f(x-\xi) d\xi d\tau + \frac{1}{b} \int_0^x \int_0^t e^{-\frac{t-r}{a}} \left[ \sum n, m(\xi, t) \right] f(x-\xi) d\xi, \\
t > \frac{c}{b} x
\end{align*}
\]  

(98)

where,

\[
\left[ \sum n, m(x, t) \right]_t = \frac{\partial}{\partial t} \left[ \sum n, m(x, t) \right] = \sum_{n=1}^{\infty} \frac{1}{n!} \left( \frac{x}{a b} \right)^n
\]
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\[ n \int_{n=2}^{\infty} \sum_{m=3}^{n} \left( -1 \right)^{n+m-1} \frac{1}{n!(m-2)!} \left( \frac{x}{b} \right)^{n-1} \frac{nt^{n-1}}{n} \]

Similarly, the matrix and air temperatures at cooling period are

\[ \vartheta_c(x,t) = \frac{1}{b} \int_0^t \int_0^x e^{-\frac{t}{b}} e^{-\frac{t}{a}} e^{-\frac{t}{a}} \left( 1 + \sum_{n,m} c \right) g(x-\xi) \]

\[ d\xi \, d\tau + g(x) e^{-\frac{t}{a}} \int_0^x \int_0^t e^{-\frac{t}{a}} \left[ 1 + \sum_{n,m} c \right] g(x-\xi) d\xi d\tau \]

\[ e^{-\frac{t}{a}} f(x-\xi) d\xi d\tau + \frac{c'}{b} \int_0^x \int_0^t e^{-\frac{t}{a}} \left[ 1 + \sum_{n,m} c \right] g(x-\xi) d\xi d\tau, \]

\[ t > \frac{c'}{b} x \]

and,

\[ \vartheta_e(x,t) = \frac{1}{b} \int_0^x e^{-\frac{t}{b}} \left[ 1 + \sum_{n,m} c \right] g(x-\xi) d\xi \]

\[ -\frac{1}{a b} \int_0^t \int_0^t e^{-\frac{t}{a}} e^{-\frac{t}{a}} \left[ 1 + \sum_{n,m} c \right] g(x-\xi) d\xi d\tau \]

\[ + \frac{c'}{b} \int_0^x e^{-\frac{t}{b}} \left[ \sum_{n,m} c \right] g(x-\xi) d\tau, \]

\[ t > \frac{c'}{b} x \]

where,

\[ \left[ \sum_{n,m} c \right] = \sum_{n=1}^{\infty} \left( -\frac{1}{n!} \right)^2 \left( \frac{xt}{a b} \right)^n + \sum_{n=2}^{\infty} \sum_{m=3}^{n} \left( -1 \right)^{n+m-1} \frac{1}{n!(m-2)!} \left( \frac{t}{a} \right)^n \left( \frac{x}{b} \right)^{m-1} \]

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and,
\[
\left[ \sum_n n, m(x,t) \right]_t' = \frac{\partial}{\partial t} \left[ \sum_n n, m(x,t) \right] = \sum_{n=1}^{\infty} \left( \frac{1}{n!} \right)^2 \left( \frac{x}{a^2 b^2} \right)^n
\]
\[
n^2 x_{n-1} + \sum_{m=n}^{\infty} \sum_{n=m}^{\infty} (-1)^{n+m-1} \frac{1}{n!(m-2)!} \left( \frac{x}{b^2} \right)^{m-1} \frac{n^{n-1}}{a^2} (102)
\]

Series of equations (97), (99) and (102) converge rapidly to the value of \( xy/ab \), and the form \( xt/ab \ \ e^{-x/ab} \) takes the small value than unity.

3. Numerical Example.

We take the example showed by A.L. London and W.M. Kaye.\(^{9,10} \) It is the case of heat exchanger for open cycle with 9:1 pressure ratio.

The chief dimensions are as follows;
- 90% intercooling, \( \Delta p/p \) \( \text{total}=2.5 \) and
- 75% regeneration.

Average properties
- Air
- Gas
Cp Kcal/kg \(^\circ\)C
- 0.247
- 0.251
kg/mh (viscosity)
- 0.104
- 0.104
Prandtl number
- 0.67
- 0.67
Specific flow rate
- 17,450 kg/1000 shp h
Capacity rate ratio
- \( Cc/Ch = 0.97 \)

From the above values we deduce the constants \( a, b \) and \( c \) as follows;
\[
a = \frac{W_r C_r M_r n}{\alpha_h A_h \tau_{oh} n} = \frac{C_r}{C_h} \frac{1}{NTU_h} \frac{1}{\tau_{oh} n}
\]
\[
b = \frac{W_h C_h}{\alpha_h A_h} \tau_{oh} = 1/NTU_h
\]
\[
c = \frac{W_h C_h}{\alpha_h A_h \tau_{oh} v_g} = \frac{L}{NTU_h C_{\tau_{oh}} v_g} (103)
\]

where, \( n \) = number of revolution of rotor, rpm
\( v_g \) = gas velocity, m/h
\( NTU_h \) = number of heat transfer unit at heating period
\( = \alpha_h A_h/W_h C_h \)

As the value \( \Sigma n, m(x,t) \) converges to \( xt/ab \) we take to the second term and considering only initial period \( f(x) = 0 \), the value of \( \partial_r (x, t) \) is approximately
\[
\partial_r (x, t) = \frac{\partial g}{\partial x} e^{-x/\alpha} \int_0^t e^{-\frac{t_\tau}{\alpha}} \left[ 1 + \sum n, m(x, \tau) \right] d\tau
\]
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\[ \frac{x}{b} e^{\frac{t}{a}} \left(1-e^{-\frac{t}{a}}\right) + \frac{x}{ab} \frac{\partial \varphi}{\partial t} e^{\frac{t}{a}} \left\{ t-a(1-e^{-\frac{t}{a}}) \right\} \]  

(104)

Now an effective period \( \tau_{oh} \) is able to deduced from the relation \( \frac{\partial \theta_r}{\partial \xi} \) \( \xi=0 \) maximum, and this result, for example, is represented in Fig.2 and effective \( \tau_{oh} \) is 0.25 minute for the case \( C_r/C_h=5.0 \) and \( NTU_h=16 \).

These values of \( \tau_{oh} \) for various \( C_r/C_h \) are graphed versus \( NTU_h \) in Fig.3. The values of \( C_{oh} \) decrease with \( NTU_h \) and increase with \( C_r/C_h \) to each same \( NTU_h \).

In practical cases, as the values of \( NTU_h \) take 5~16, we can consider the effective period \( C_{oh} \) ranges from 0.1 to 2 minutes for matrix type regenerator.

In Fig.4 the relations between \( NTU_h \) and \( n \) rpm are shown for the case of \( C_r/C_h=5.0 \) and \( NTU_h=16 \), and as \( n \) gets larger the value of \( C_{oh} \) decreases logarithmically and takes about 0.35 minute in the case \( n=50 \) rpm.\(^{(11)}\)

In equation (103), the value of \( c \) takes different one refering to the gas velocity \( v_g \) m/s, and for the case of \( L=1.04m \) the value of \( c/b \) versus \( C_{oh} \) is shown in Fig.5, and in Fig.6 \( c/b \) is graphed to \( v_g \) for the case \( C_{oh}=0.31 \) min.
On each cases the value of $c/b$ is very small comparing with the other terms, and almost negligible one.

In the equations referring to $\theta_r$ and $\theta_n$, the condition of step function being zero is $0 < t < c/b \quad x = x_0 \frac{L_0}{C_{n}}$ and this relation is represented in Fig.7 for the various $v_o$.

From the graphs we can consider that this condition is out of question for the cases of small $t/\tau_{oh}$ and $x/L$.

From these conditions and relations, we find the temperature distributions, for example, the case that $C_c/C_h=5.0$, $n=50$ rpm, $NTU_h=13$ and $C_{oh}=0.4$ min. and shown in Fig.8.

In Fig.9 the relations between temperature of first period and steady state are shown, and from this graph we can understand that the temperature in steady state takes about 23% higher value than that of the first period, therefore in Fig.8 the effectiveness takes about 85% for $C_r/C_h=5.0$.

On the same example as in Fig.10, temperatures $\theta_r/\theta_o$ and $\theta_n/\theta_o$ are plotted during heating and cooling periods, i.e., one cycle.

In Fig.11 and 12 the effectiveness for counter and parallel flow respectively, where $C_c/C_h=1.0$ and $(\alpha A)^o=\alpha_c A_c/\alpha_h A_h=1.0$. 
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Fig. 6

Fig. 7
In the last heating period the temperature $\theta_1/\theta_2$ is represented approximately for the case of counter flow as follows.
\[
\frac{\partial \phi}{\partial \theta} \bigg|_{x=0} - \frac{1}{e} = \frac{f(x)}{x} = \partial_x \left( 1 - \frac{1}{e} \right) + \frac{fe(0)}{e} \cdot \frac{1}{a} 
\]

where, \( f_e(0) \) is effective matrix temperature.

And exit temperature at heating period is approximately

\[
\frac{\partial \phi}{\partial \theta} \bigg|_{x=1} \approx \left[ \left( 1 - e^{-\frac{1}{a}} \right) + \frac{fe(0)}{e} \cdot \frac{1}{a} \right] \left[ \left( 1 - e^{-\frac{1}{b}} \right) - \frac{e}{\beta} \cdot \frac{1}{b} \left( 1 - e^{-\frac{t}{a}} \right) \right]
\]

Therefore the effectiveness becomes

\[
\mathcal{E} \text{ counter} = \frac{1}{e} \int_{0}^{\tau_{oe}} \left. \frac{\partial \phi}{\partial \theta} \right|_{x=0} dt \approx \left[ \left( 1 - e^{-\frac{1}{a}} \right) + \frac{fe(0)}{e} \cdot \frac{1}{a} \right] \\
\times \left[ \left( 1 - e^{-\frac{1}{b}} \right) - \frac{e}{\beta} \cdot \frac{1}{b} \left( 1 - e^{-\frac{1}{a}} \right) \right]
\]

Similarly, for parallel flow

\[
\mathcal{E} \text{ parallel} = \frac{1}{e} \int_{0}^{\tau_{oe}} \left. \frac{\partial \phi}{\partial \theta} \right|_{x=0} dt
\]

\[
\approx \left[ e^{-\frac{1}{b}} \left( 2 - e^{-\frac{1}{a}} + ae^{-\frac{1}{a}} \right) + \frac{g(0)}{e} \left\{ e^{-\frac{1}{a}} \left( 1 + \frac{1}{a} \right) - \frac{e}{b} \cdot \frac{1}{b} \right\} + 1
\]

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\[-e^{-\frac{1}{b'}} \left\{ \left(1-e^{-\frac{1}{b'}}\right)- \frac{1}{b'} \left(1-e^{-\frac{1}{a'}}\right) a'+1 \right\}\]  \hspace{1cm} (108)

where, \(a', b'\) and \(c'\) are the constants in cooling period.

In Fig.11 the result calculated from eq. (107) agrees with the H. Hausen’s result within about 1% error for \(C_v/C_s = 1.0\)

Conclusion.

In this paper the theoretical research for the regenerative heat exchanger was performed and the writer succeeded in finding the exact solution by using two-dimensional Laplace-Transformation.

The writer’s dual operational method is also very effective one to solve the heat exchanger problems.

For the numerical calculation, the solution with serial form is the most convenient and efficient method, and in the last section the practical example is treated, namely, effective period, temperature distributions and effectivenesses are graphed.

In the end the writer wishes to express hearty thanks to Dr. Prof. Y. Tanasawa of Tohoku Univ. for his valuable suggestions.

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伝熱問題におけるラプラス変換の応用例（その4）

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今回は、熱交換器として最も一般性のある微分方程式を有するマトリックス型熱式の場合について解析を試み、2次元変換を用いてその解析に成功した。この型の特殊の場合として従来の直流流流形およびプレートフイン形、熱交換器の結果が導き出される。数値計算については今後の様に逆数法を用い、有効周期の概念より温度分布、再生率などを求め図示した。この結果は「伝熱問題におけるラプラス変換の応用」および「ガスタービン用熱交換器の計算について」と題して昭和30年9月9日応用力学連合講演会（東京）および同年7月9日機械学会東北地方講演会（米沢）において発表をした。

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