On Generalized Coupling Coefficient k.

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§0. Introduction

Any two circuits so arranged that energy can be transferred from one to the other are often called a coupling network, even though this transfer of energy takes place by some means such as a condenser, resistance, or inductance common to the two circuits rather than by the aid of a mutual inductance. And the network's coupling coefficient k represents the degree of electrical proximity between one circuit and the other.

In general, 4-terminal network expressed by using k may be considered as a coupling network. In the present paper, we try to analyse any 4-terminal network by the aid of the generalized (1) coupling coefficient k.

In §1, we arrange and explain the fundamental equations on general 4-terminal coupling network.

In §2, we take the network's calculation examples (i.e. determination of 4-terminal constants and the balanced condition of the impedance bridge and others).

and §3, §4 contain some general remarks on the network analysis and calculations (i.e. back-coupling oscillation circuits and filter circuits).

By this investgation it is seen that the calculations by using k is useful in any network analysis and make its physical meaning clear.

§1. Basic 4-terminal network equation considered as coupling network.

Only the linear passive network with lumped constants is treated in this paper. In Fig.1, if the elements of the general 4-terminal network are chosen as follows:

N : a given linear passive network.

points 1—1′ 2—2′ : source (or input) and load (or output) terminals.

E, I : terminal voltage and current.

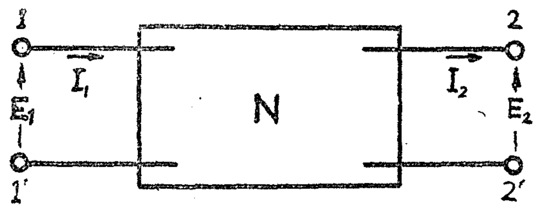


Fig.1 General Linear Passive 4-terminal Network

Z_{11}, Z_{22} : the open-circuit input impedances (or open circuit driving point impedances) with respect to terminals 1–1′ and 2–2′ respectively.

(i. e. Z_{11} is the impedance looking into terminals 1–1′ with terminals 2–2′ open, similarly, Z_{22} is the impedance looking into terminals 2–2′

with terminals 1-1′ open).

Z_{12}, Z_{21} : the open-circuit mutual (or transfer) impedance with respect to terminals 1—1′ and 2—2′.

Y_{11}, Y_{22} : the short-circuit input admittances (or short-circuit driving-point admittances) with respect to terminals 1—1′ and 2—2′.

Y_{12}, Y_{21} : the short-circuit mutual (or transfer) admittance with respect to terminals 1—1′ and 2—2′.

Then the basic relations are as follows :

1 . Z—matrix representation

$$\begin{vmatrix} E_1 \\ E_2 \end{vmatrix} = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} \begin{vmatrix} I_1 \\ I_2 \end{vmatrix} \quad \text{and} \quad Z_{12}=Z_{21}=Z_m \tag{1.1}$$

Y—matrix representation

$$\begin{vmatrix} I_1 \\ I_2 \end{vmatrix} = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} \begin{vmatrix} E_1 \\ E_2 \end{vmatrix} \quad \text{and} \quad Y_{12}=Y_{21}=Y_m \tag{1.2}$$

3 K—matrix representation

$$\begin{vmatrix} E_1 \\ I_1 \end{vmatrix} = \begin{vmatrix} A_1 & B \\ C & A_2 \end{vmatrix} \begin{vmatrix} E_2 \\ I_2 \end{vmatrix} \quad \text{and} \quad A_1A_2-BC=1 \tag{1.3}$$

(A_1, A_2, B, C are 4—terminal constants)

If terminals 2—2′ are short—circuited ($E_2=0$) and 2—2′are open—circuited ($I_2=0$), Eq. (1.1) and (1.2) are as follows.

$$\left.\frac{E_1}{I_1}\right|_{\text{at } E_2=0} = Z_{11}\left(1-\frac{Z_m^2}{Z_{11}Z_{22}}\right), \qquad \left.\frac{I_1}{E_1}\right|_{\text{at } I_2=0} = Y_{11}\left(1-\frac{Y_m^2}{Y_{11}Y_{22}}\right) \tag{1.4}$$

Then, by the application of the theorem of coupling networt, k becomes

$$k^2=\frac{Z_m{}^2}{Z_{11}Z_{22}}=\frac{Y_m{}^2}{Y_{11}Y_{22}} \tag{1.5}$$

Further, the short and open -circuit impedances are represented from the relation of matrix (1), (2) and (3) in the following way, containing k.

$$Z_{1S}=\left.\frac{E_1}{I_1}\right|_{\text{at } E_2=0} = Z_{11}\left(1-\frac{Z_m^2}{Z_{11}Z_{22}}\right)=Z_{11}(1-k^2)=\frac{B}{A_2}$$

$$Z_{1f}=\left.\frac{E_1}{I_1}\right|_{\text{at } I_2=0} = \frac{1}{Y_{11}\left(1-\frac{Y_m^2}{Y_{11}Y_{22}}\right)}=\frac{1}{Y_{11}(1-k^2)}=\frac{A_1}{C}$$

$$Z_{2S}=\left.\frac{E_2}{I_2}\right|_{\text{at } E_1=0} = Z_{22}\left(1-\frac{Z_m^2}{Z_{11}Z_{22}}\right)=Z_{22}(1-k^2)=\frac{B}{A_1}$$

$$\left.\begin{aligned} Z_{2f}&=\frac{E_2}{I_2}=\frac{1}{Y_{22}\left(1-\frac{Y_m^2}{Y_{11}Y_{22}}\right)}=\frac{1}{Y_{22}(1-k^2)}=\frac{A_2}{C} \\ &\text{at } I_1=0 \\ \text{and} \\ Z_{12}&=Z_{21}=Z_m=\frac{1}{C} \\ Y_{12}&=Y_{21}=Y_m=\frac{1}{B} \end{aligned}\right\} \quad (1.6)$$

Here,

Z_{1S}=self-impedance looking into terminals 1—1′ with 2—2′ short-circuited.
Z_{1f}=self-impedance looking into terminals 1—1′ with 2—2′ open-circuited.
Z_{2S}=self-impedance looking into terminals 2—2′ with 1—1′ short-circuited.
Z_{2f}=self-impedance looking into terminals 2—2′ with 1—1′ open-circuited.

From the above results,if we give the 4-terminal constants by using matrix (3)

$$\left.\begin{aligned} \begin{vmatrix} E_1 \\ I_1 \end{vmatrix}=\begin{vmatrix} A_1 & B \\ C & A_2 \end{vmatrix}\begin{vmatrix} E_2 \\ I_2 \end{vmatrix}&=\begin{vmatrix} \frac{Z_{1f}}{Z_m} & Z_m\left(\frac{1}{k^2}-1\right) \\ \frac{1}{Z_m} & \frac{Z_{2f}}{Z_m} \end{vmatrix}\begin{vmatrix} E_2 \\ I_2 \end{vmatrix} \\ &=\begin{vmatrix} \frac{Y_{2S}}{Y_m} & \frac{1}{Y_m} \\ Y_m\left(\frac{1}{k^2}-1\right) & \frac{Y_{1S}}{Y_m} \end{vmatrix}\begin{vmatrix} E_2 \\ I_2 \end{vmatrix} \end{aligned}\right\} \quad (1.7)$$

So this is a representation of K-matrix type and the propagation constant (θ) of a network has the following relations.

$$\left.\begin{aligned} k^2&=\frac{Z_m^2}{Z_{1f}Z_{2f}}=\frac{Z_m^2}{Z_{11}Z_{22}} \\ k^2&=Y_m^2 Z_{1S}Z_{2S}=\frac{Y_m^2}{Y_{11}Y_{22}} \\ \text{and} \\ k^2&=\frac{1}{A_1A_2}=1-\frac{Z_s}{Z_f}=1-\tanh^2\theta \end{aligned}\right\} \quad (1.8)$$

$$\frac{Z_{1S}}{Z_{1f}}=\frac{Z_{2S}}{Z_{2f}}=\frac{Z_S}{Z_f}=\frac{BC}{A_1A_2}=\tanh^2\theta. \quad , \theta=\beta+j\alpha$$

(β : attenuation constant, α : phase constant)

In addition to the above,

$$\cosh\theta=\sqrt{A_1A_2}=\frac{1}{k}, \ \sinh\theta=\sqrt{BC}=\sqrt{\frac{1}{k^2}-1}, \ \tanh\theta=\sqrt{1-k^2} \quad (1.9)$$

Next, from Eq. (1.7) image impedances Z_{01}, Z_{02} are as follows.

$$\left.\begin{aligned} Z_{01}&=\sqrt{\frac{BA_1}{CA_2}}=Z_{1f}\sqrt{1-k^2}=\frac{Z_{1S}}{\sqrt{1-k^2}} \\ Z_{02}&=\sqrt{\frac{BA_2}{CA_1}}=Z_{2f}\sqrt{1-k^2}=\frac{Z_{2S}}{\sqrt{1-k^2}} \end{aligned}\right\} \quad (1.10)$$

In case of the symmetrical network.

$$Z_0=\sqrt{Z_{01}Z_{02}} \quad (1.11)$$

k shown here is, of cource, a vector. So it is important to note that this k, the coupling coefficient, is of the form $A\angle\theta$ where both the magnitude A and the angle θ are changing with frequency. Let us attempt the analysis as the coupling network to any given 4-terminal network, and we call this method temporalily the coupling parameter.

§2. Circuit's calculation by using the coupling parameter

Here, we shall deal with some unsymmetrical circuits. Explanation will be made on some examples of calculation.

(A) Determination of 4-terminal constants: In general, the determination of 4-terminal constants of a circuit is important for detailed analysis of 4-terminal network. The ways usually used belong, after all, to the following two.

(i) Application of the Kirchhoff's Laws to the network (i.e. the constant can be derived from the relations of the input and output terminal vol tages and currents). Further, in case of the network connected incascade, the application of matrix calculus to the network is convenient to determine the constants of network.

(ii) Application of the short and opem circnit impedances of network (i. e. the constants can be derived from the relation of the both impedances and $A_1A_2-BC=1$). In this case, we need not take voltage and current into considerat-ion.

As the calculation by the aid of the coupling parameter described in this paper is determined only by the network's construction, the method naturally belongs to section (ii).

The following are some merits of this method.

The determination of k is only due to the open and short-circuit impedances of one side terminals(1—1′ or 2—2′). By a pair terminal impedance, a measure of total coupling of the circuit is given.

Further, the determination of Z_m of any circuit is very easy, because Z_m can be calculated immediately from the construction of circuits, as mentioned in the Table (except the case that there are some mutual impedances between the circuit's elem-ents), that is

In the single-circuit which contains no loop (e. g. ᒣ (L), T I, -type networks etc),

Z_m is the parallel -type impedance to terminals 1—1′, and 2—2′, that is, Z_b in

˥-type, Z_c in T-type, Z_e in I-type, $\frac{Z_aZ_b}{Z_a+Z_b+Z_c}+Z_d$ In ╤-type etc.

In the complex-circuit which contains some loops (e.g, Π, D. Π, X, type networks etc.)

The denominator of Z_m is the sum of the total impedances of the loop, that is, $Z_a+Z_b+Z_c$ in Π, Π-type, $Z_a+Z_b+Z_c+Z_d$ in D, X-type, $Z_a+Z_b+\frac{Z_cZ_d}{Z_c+Z_d}$ in ⊟-tyee etc.

The numerator of Z_m is the product of the parallel-type impedance, that is Z_bZ_c in Π, D-type, Z_aZ_b in Π-type, $Z_aZ_cZ_d$ in Π-type, $Z_bZ_c-Z_aZ_d$ inX-type etc.

After all, it can be seen that, 4-terminal constants are easily determined with the use of coupling parameter especially in complex network.

(B) On circuit calculations: In many cases of circuit calculations (e.g. the determination of the balance of impedance bridge), it is convenient to treat a given circuit as a coupling networt. Here, the magnitude of k can bear some relation to the physical meaning of network from Eq. (1.9)

That is, when the network is in close coupling condition (k is large), energy with respect to terminals 1—1′ is transmitted to terminals 2—2′ with little leakage, and when the network is in loose coupling condition, much leakage is observable. Now, let us suppose that the closest coupling means $|k|=1$ and loosest coupling means $|k|=0$. Under this supposition the variations of network elements are indicated in the Table.

Therefore, for instance, from the Table, it is evident that the determination of the balance of impedance bridge coincides with the condition of lattice-type network where $|k|=0$. Because, the denominator of k is zero when $Z_aZ_d=Z_bZ_c$.

And, in this case, k is obtainable by a much simpler procedure.

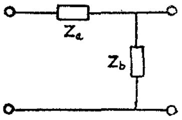
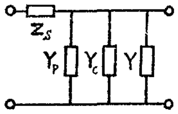
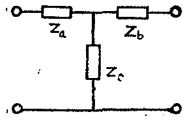
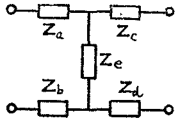
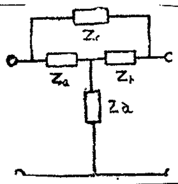
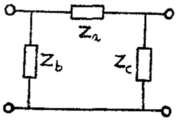
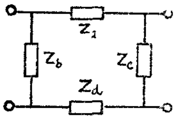
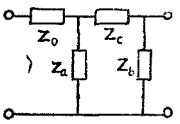
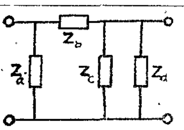
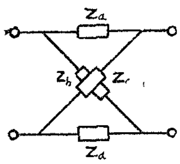
And the analysis of any back-coupling oscillation circuit which will be explained int he next chapter becomes very easy when the oscillation circuit is considered as one of the coupling 4—terminal networks.

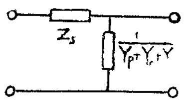
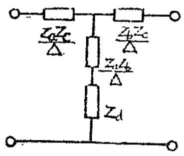
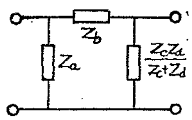
Further, in filter circuit, it is found that the generalized coupling coefficient k is very useful to the circuit analysis if we suppose that k may be any number real imaginary or complex.

§3. On back-coupling oscillation circuit

In case of oscillation circuit analysis, it is usual to solve the general differential equation of oscillation expressed by the instantaneous values of voltage and current. Especially the determination of oscillatory condition and frequency in steady state may be got from the solution of equation represented by vector notation, under the supposition that applied voltage and current are of pure sine wave form.

Then, as the oscillatory circuit can be considered, as mentioned later, as one of the coupling 4—terminal networks, we analyse the oscillatory system in this chapter

Type	Construction of Network	Mutual Impedance Z_m	Coupling-Coefficient k	K--matrix $A_1=\frac{Z_{1f}}{Z_m}$ $B=Z_m\left(\frac{1}{k}-1\right)$ $C=\frac{1}{Z_m}$ $A_2=\frac{Z_{2f}}{Z_m}$
┐ (L)		Z_b	$\sqrt{\frac{Z_b}{Z_a+Z_b}}$	$\begin{bmatrix} 1+\frac{Z_a}{Z_b} & Z_a \\ \frac{1}{Z_b} & 1 \end{bmatrix}$
Ш		$\frac{1}{Y+Y_p+Y_c}$	$\frac{1}{\sqrt{Z_s(Y_p+Y_c+Y)+1}}$	$\begin{bmatrix} (1+Z_sY_c)+Z_s(Y_p+Y) & Z_s \\ Y_p+Y_c+Y & 1 \end{bmatrix}$
T		Z_c	$\frac{Z_c}{\sqrt{(Z_a+Z_c)(Z_b+Z_c)}}$	$\begin{bmatrix} 1+\frac{Z_a}{Z_c} & Z_a+Z^b+\frac{Z_aZ_b}{Z_c} \\ \frac{1}{Z_c} & 1+\frac{Z_b}{Z_c} \end{bmatrix}$
I		Z_e	$\frac{Z_e}{\sqrt{(Z_a+Z_b+Z_e)(Z_c+Z_d+Z_e)}}$	$\begin{bmatrix} 1+\frac{Z_a+Z_b}{Z_e} & \frac{Z_e(\triangle+Z_d)+(Z_a+Z_b)(Z_c^{?}+Z_d)}{Z_e} \\ \frac{1}{Z_e} & 1+\frac{Z_c+Z_d}{Z_e} \end{bmatrix}$
〒		$\frac{\square}{\triangle}$	$\frac{\square}{\sqrt{(\square+Z_bZ_c)(\square+Z_aZ_c)}}$	$\begin{bmatrix} 1+\frac{Z_aZ_c}{\square} & \frac{Z_aZ_bZ_cZ_d}{\square}\left(\frac{1}{Z_a}+\frac{1}{Z_b}+\frac{1}{Z_d}\right) \\ \frac{\triangle}{\square} & 1+\frac{Z_bZ_c}{} \end{bmatrix}$
π		$\frac{Z_bZ_c}{\triangle}$	$\sqrt{\frac{Z_bZ_c}{(Z_a+Z_b)(Z_a+Z_c)}}$	$\begin{bmatrix} 1+\frac{Z_a}{Z_c} & Z_a \\ \frac{\triangle}{Z_bZ_c} & 1+\frac{Z_a}{Z_b} \end{bmatrix}$
D		$\frac{Z_bZ_c}{\triangle+Z_d}$	$\sqrt{\frac{Z_bZ_c}{(Z_a+Z_b+Z_d)(Z_a+Z_c+Z_d)}}$	$\begin{bmatrix} \frac{Z_a+Z_c+Z_d}{Z_c} & Z_a+Z_d \\ \frac{\triangle+Z_d}{Z_bZ_c} & \frac{Z_a+Z_b+Z_d}{Z_b} \end{bmatrix}$
Π		$\frac{Z_aZ_b}{\triangle}$	$\frac{Z_a\sqrt{Z_b}}{\sqrt{(Z_a+Z_c)\{Z_0\triangle+Z_a(Z_b+Z_c)\}}}$	$\begin{bmatrix} \frac{Z_0\triangle+Z_a(Z_b+Z_c)}{Z_aZ_b} & \frac{Z_0(Z_a+Z_c)+Z_aZ_c}{Z^a} \\ \frac{\triangle}{Z_aZ_b} & 1+\frac{Z_c}{Z_a} \end{bmatrix}$
Ш		$\frac{Z_aZ_cZ_d}{(Z_a+Z_b)(Z_c+Z_d)+Z_cZ_a}$	$\sqrt{\frac{Z_aZ_bZ_d}{(Z_a+Z_b)\{Z_b(Z_c+Z^d)+Z_cZ_d\}}}$	$\begin{bmatrix} 1+\frac{Z_b(Z_c+Z_d)}{Z_cZ_d} & Z_b \\ \frac{(Z_a+Z_b)(Z_c+Z_d)+Z_cZ_d}{Z_aZ_cZ_d} & 1+\frac{Z_b}{Z_a} \end{bmatrix}$
X		$\frac{Z_bZ_c-Z_aZ_d}{\triangle+Z_d}$	$\frac{Z_bZ_c-Z_aZ_d}{\sqrt{(Z_a+Z_b)(Z_c+Z_d)(Z_a+Z_c)(Z_b+Z_d)}}$	$\begin{bmatrix} \frac{(Z_a+Z_c)(Z_b+Z_d)}{Z_bZ_c-Z_aZ_d} & \frac{Z_aZ_c(Z_b+Z_d)+Z_bZ_d(Z_a+Z_c)}{Z_bZ_c-Z_aZ_d} \\ \frac{\triangle+Z_d}{Z_bZ_c-Z_dZ_a} & \frac{(Z_a+Z_b)(Z+Z_d)}{Z_bZ_c-Z_aZ} \end{bmatrix}$
		$\triangle=Z_a+Z_b+Z_c$ $\square=Z_d\triangle+Z_aZ_b$	$k=\sqrt{1-\frac{Z_s}{Z_f}}=\sqrt{1-\frac{Y_f}{Y_s}}$	or $A_1=\frac{Y_{2s}}{Y_m}$ $B=\frac{1}{Y_m}$ $C=Y_m\left(\frac{1}{k^2}-1\right)$ $A_2=\frac{Y_{1s}}{Y_m}$

Properties of Networks Under the variation of the magnitude of k		Notes	
$\|k\|=1$	$\|k\|=0$		
$Z_a=0$ (Series-type Impedsnce) or $\begin{pmatrix} Z_a = \text{Series Resonance} \\ Z_b = \text{Parellel Resonance} \end{pmatrix}$	$Z_b=0$ (Parallel-type Impedance) or $\begin{pmatrix} Z_b = \text{Series Resonance} \\ Z_a = \text{Parallel Resonance} \end{pmatrix}$	Filter Circuit (Mid-half Section) Attenuation Equalizer	
$Z_\varepsilon = -\dfrac{1}{Y_p+Y_c+Y}$	$\dfrac{1}{Y_p+Y_c+Y}=0 \quad (Z_pZ_cZ=0)$	(Equivalent ┐-Circuit)	
$Z_a=-\dfrac{Z_bZ_c}{Z_b+Z_c}$	$Z_c=0$	Filter Circuits	
$Z_a+Z_b=-\dfrac{Z_e(Z_c+Z_a)}{Z_c+Z_d+Z_e}$	$Z_e=0$	Filter Circuit Attenuating Circuit	
$Z_d=-\dfrac{Z_aZ_b}{Z_a+Z_b}$	[illegible] $=0$	Filter Circuit Attenuating Circuit Phase Corrector Circuit (Bridged-T-type)	
$Z_a=-(Z_b+Z_c)$	$Z_bZ_c=0$	Filter Circuit	
$Z_a=-Z_d$	$Z_bZ_c=0$	Filter Circuit Attenuating Circuit	
$Z_0=-\dfrac{Z_aZ_c}{Z_a+Z_c} \quad (\Delta\neq0)$	$Z_aZ_b=0$	Oscillatory Circuit (back-coupling)	
$Z_a=-\left(Z_b+\dfrac{Z_cZ_d}{Z_c+Z_d}\right)$	$Z_aZ_bZ_d=0$	(Equivalent π-Circuit)	
$Z_a+Z_b=0,\ Z_c+Z_d=0$ the condition of the constant-current bridged-network.	$Z_aZ_d=Z_bZ_c$ the general equation of balance for the impedance bridge	Filter Circuit Attenuation Fqualizer Phase Corrector Circuit (Lattice-type)	
When $\|k\|=1$, Z_{1s} is equal to 0.	When $\|k\|=0$, Z_{1s} is equal to Z_{1f}.		

as a spécial type of coupling network.

In Fig. 1, any current or voltage (e.g. E_2) appearing in terminals 2—2′, must be the result of the application of source (E_1) to terminals 1—1′.

Now when these voltages are represented by the vecter notation the gain of voltage amplification is as follows.

$$|A| = \left|\frac{E_2}{E_1}\right| \tag{3.1}$$

Here, $|A|$, A are respectively voltage gain and gain vector.
And A is equal to inverse A_1 and because of its utility in circuit analysis, the variation of A which changes with frequency is of considerable importance.[2]

Now it is necessary that the property of phase-shift oscillation is connected with the above condition. If we treat a given oscillatory circuit as a coupling 4—terminal network and find Z_m and k of the circuit, we can easily find the relation of circuit's elments that satisfies the oscillatory condition.

Fundamental circuit (Single-tube circuit):

We shall deal with the oscillatory circuit using a triode tube only in this section. Fig.2 (a) shows a typical back-coupling circuit. Here, let it be suppcsed that the grid of the tube has no current. Then, even if the circuit is opened as in Fig. (2) (b), the circuit's condition is not changed. So, as shown in (c), a given circuit is transformed into a coupling 4—terminal -type network. (refer. the Table)

Next, we shall consider about the phase-shift action; that is, the amplification factor μ becomes $-\mu$ as a result of the phase-shift action in case of a single tube. If any circuit whose attenuation and phase-shift constant are $\frac{1}{\mu}$ and $-\frac{1}{\mu}$ respectively, is connected with a tube, this circuit as a whole becomes $A=(-u)\left(-\frac{1}{u}\right)=1$. So the oscillatory condition is

$$A=1 \tag{3.2}$$

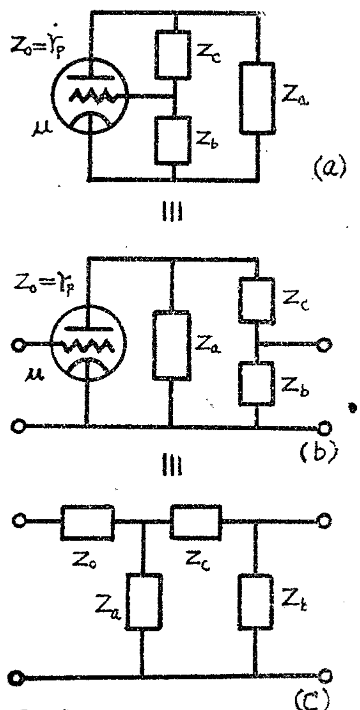


Fig. 2 Back-coupling Oscillatory Network (single-Tube)

If we consider only the voltage ratio E_2/E_1, except the phase-shift constant under the conditio that the grid current is zero, $\frac{1}{A_1}=\frac{E_2}{E_1}$.

From Eq. (3.1) (3.2), when A_1 becomes unity, the circuit keeps on oscillating. So we arrange the fundamental equations of oscillatory condition as follows: (refer

Eq. (1.7) (1.8) and the Table)

$$\left.\begin{array}{ll} & A_1=1(u_0) \\ & Z_{1f}=Z_m(u_0) \\ \text{or} & (\mu_0)\ k=\sqrt{\frac{A_1}{A_2}}\left(=\sqrt{\frac{Z_{1f}}{Z_{2f}}}=\sqrt{\frac{Z_{1S}}{Z_{2S}}}\right) \end{array}\right\} \quad (3.3)$$

μ_0: phase-shift constant

Then, by using any equation of Eq. (3.3), the oscillatory condition gives

$$(\mu_0)Z_aZ_b=Z_0(Z_a+Z_b+Z_c)+Z_a(Z_b+Z_c) \quad (3.4)$$

But Eq. (3.4) is introduced when no mutual impedance is found between the circuit elements.

And when Eq. (3.4) is satisfied, the value of k becomes as follows, except the phase-shift constant.

$$k^2=\frac{1}{1+\frac{Z_c}{Z_a}} \quad (3.5)$$

Here, in oscillatory condition, $|Z_a+Z_b|\simeq Z_c$ and Z_a, Z_c are always imepdances of opposite signs. So the magnitude of k becomes larger than unity in some cases. (e. g. in resonance state)

In many practical cases the elements of circuit can be changed according to the sorts of the network, for example, in the Hartley circuit $Z_a=\frac{1}{j\omega C^a}$, $Z_b=\frac{1}{j\omega C_b}$, $Z_c=\gamma+j\omega L$, $Z_0=\gamma_p, \mu_0=-\mu$.

In addition, as the plate-tuning type and the grid-tuning type oscillatory circuits can be transformed into a fundamental coupling 4—terminal network in Fig.2(a), Eq.(3.4) is applied to these circuits too.

Two-tubes and multi-tubes circuit:

In this case, we consider a given circuit as a circuit which is connected in cascade, then

$$A=A_1A_2\cdots\cdots A_n=1 \quad (3.6)$$

After all, as mentioned above, the solution of Eq. (3.6) gives the oscillatory condition for multi-tubes circuit,

But in this case the modes of oscillation which change with the phase-shift constant are omitted here. (3)

§4. Application to wave filter.

In general, wave filter is one of the continuous networks. Usually each unit element of the network is of equal construction and is connected by the same method. The main properties of wave filter are as follows. In ideal networks:

(a) passing band (P.B.) A frequency band has zero attenuation. From. Eq (1.8), the attenuation constant $\beta=0$, and the image impedance is real in this band.

(b) Attenuting band (A.B.) A frequency band has not zero attenuation..

As mentioned above, $\beta \neq 0$ and the image impedance is imaginary.

(c) Cut-off frequency (c.F.) The frequencies which separate the above two bands are called the cut-off or critical frequencies.

In practical networks :

The theoretical network should be constructed with pure reactance elements and the practical network must be of a small attenuation inside these bands because of unavoidable coil resistance and condenser losses; but the properties of frequency discrimination are so much greater than those of simple tuning devices that the practical forms may with justice and convenience be called wave filters.

Remembering the above proprties, we shall study the wave filters by the aid of the generalized coupling coefficient k. Here we will deal with some ideal filter networks.

(1) P.B

From (a), $\beta = 0$ $\quad \therefore \theta = j\alpha$

In P.B, therefore, the follow ing conditions are given.

$$\left.\begin{aligned} &\tanh\theta = j\tan\alpha = \sqrt{1-k^2} = \sqrt{\frac{Z_s}{Z_f}} \\ &k^2 = \frac{1}{\cosh^2\theta} = \frac{1}{\cos^2\alpha} = 1 - \frac{Z_s}{Z_f} \\ &Z_0 = \sqrt{Z_s Z_f} \end{aligned}\right\} \qquad (4,1)$$

In order that Eq, (4,1) is satisfied

(i) It is necessary that Zs and Z_f are reactances of opposite sign.

(ii) k can be of positive or negative real number, but from Eq. (4.1.), $0 \leq \cos^2\alpha \leq 1$, so *the critical value is $k = \pm 1$.*

(iii) when α is $m\pi$, then $\cos\alpha = \pm 1$, $k = -1$ and α becomes $(2n+1)\pi$ in one p.B. (m,n are zero or any integer)

After all, in P.B, k must lie between -1 and $+1$

$$0 \leq \frac{1}{k^2} \leq 1 \qquad (4.2)$$

and the image impedance Z_0 is as follows.

$$Z_0 = \text{real quantity (resistve component only)} \qquad (4.3)$$

(2) A.B

From (b), $\beta \neq 0$, so *k can be any number eXcept* (4.2)

$$\frac{1}{k^2} \geq 1 \text{ and } \frac{1}{k^2} \leq 0$$

From Eq. (4.1), as mentioned above,

(i) It is necessary that Z_S and Z_f are reactances of the same sign

For $\frac{1}{k^2} \geq 1$, $\quad Z_f > Z_f - Z_S|$ and Z_S, Z_f are of the same sign.

For $\frac{1}{k^2}\leq 0$, Z_f, $(Z_f - Z_S)$ are of the opposite signs.

(ii) $\tanh\theta=\sqrt{1-k^2}$ becomes real number and in order that tanh θ is real,

$\alpha=\frac{m\pi}{2}$,

Here, $\left(\tanh\theta=\tanh(\beta+j\alpha)=\frac{\tanh\beta+j\tan\alpha}{1+j\tanh\beta\tan\alpha}\right)$

(A.B.—I)

$\tanh\theta=\tanh B<1$ (m=even number)

$$k^2=1-\tanh^2\beta\leq 1 \qquad \left(\alpha=\frac{2n\pi}{2}\right) \tag{4.5}$$

k is real and when $\beta=\infty$, k is zero and when $\beta=0$, k^2 is equal to unity. So the critical value of the two bands is $k=\pm 1$.

(A.B.—II)

$\tanh\theta=\coth\beta>1$ (m=odd number)

$$k^2=1-\coth^2\beta\leq 0 \qquad \left(\alpha=\frac{(2n+1)\pi}{2}\right) \tag{4.6}$$

k is imaginary and when $\beta=0$, k^2 is equal to $-\infty$, and when $\beta=\infty$, k is equal to zero. So the critical vale is $k=\pm j\infty$.

And the image impedance Z_0 is as follows.

$$Z_0=\text{imaginary quantity (reactive component only)} \tag{4.7}$$

Fundamental circuit (L-tyqe)

In general, the simple form of a continuous network is a ladder, a latice or a bridged-T type. It can be devided into two unsymmetrical ┐(L)-sections, as shown in Fig.3.

Now we will consider this ┐(L)-section circuit as a fundamental circuit and study its characteristics.

From the Table, the given circuit's k^2 becomes $\frac{4Z_2}{Z_1+4Z_2}$.

Here, if we substitute $K=\frac{Z_1}{4Z_2}$, as mentioned above, the whole circuit's characteristics are as follows.

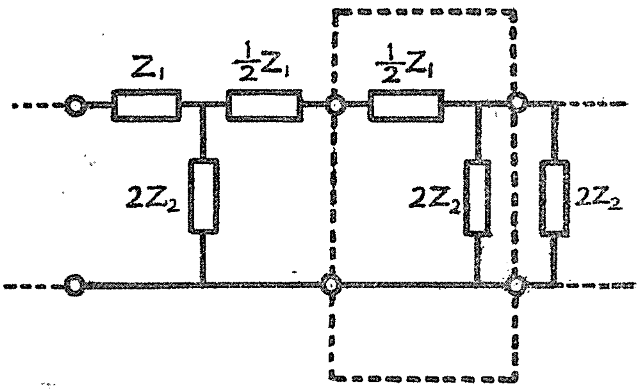


Fig.3 T and ┐ sections (fundamental circuit)

(1) P. B.

$$0\leq\frac{1}{k^2}\leq 1 \quad (-1\leq K\leq 0) \qquad \therefore\ K=\frac{Z_1}{4Z_2}\geq -1,\quad K=\frac{Z_1}{4Z_2}\leq 0$$

Z_1 and Z_2 are reactances of opposite signs.

and $|Z_1| \leq |4Z_2|$

(2) A. B.

$$1 \leq \frac{1}{k^2} \leq 0 \qquad (0 \leq K \leq -1)$$

So

(A.B.-Ⅰ) $k^2 \leq 1 \;\therefore\; K = \frac{Z_1}{4Z_2} > 0 \qquad \alpha = \frac{2n\pi}{2}$

Z_1 and Z_2 are reactances of same signs.

(A.B.-Ⅱ) $k^2 < 0 \;\therefore\; K = \frac{Z_1}{4Z_2} \leq -1 \qquad \alpha = \frac{(2n+1)\pi}{2}$

Z_1 and Z_2 are reactances of opposite signs.

and $|Z_1| > |4Z_2|$

(3) C. F. C.F. is given by $k = \pm 1$.

General circuit (I.D.-type)

If $Z = j\omega L$, $Y = j\omega C$, from I, D-type netwoyk in the Table, k and θ of the two circuits are indicated as follows.

$$k = \frac{2}{2 - \omega^2 cL}, \quad \cosh\theta = \frac{1}{k}$$

(i) C. F. The cut-off frequency is a frequency when $k^2 = 1$; therefore

$$\omega_0 = \frac{2}{\sqrt{LC}}$$

(ii) Filter characteristics

As we can consider these circuits as symmetrical networks these circuits can be devided into the above fundamental circuits (refer. Fig. 3)

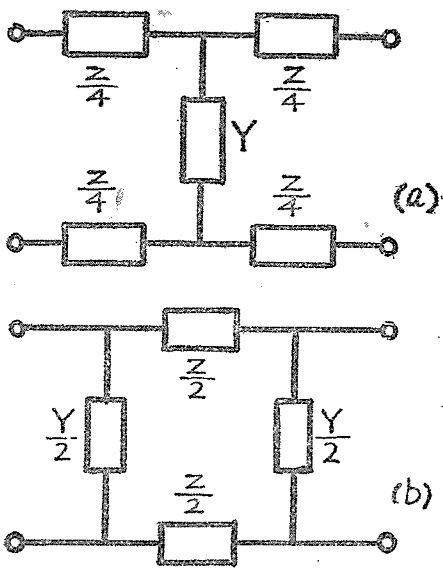


Fig.4 I and D Sections

So the coupling parameter K of the fundamental circuit, from $\sinh\frac{\theta}{2} = \sqrt{\frac{1}{2}(\cosh\theta - 1)}$

$$K = \frac{1-k}{2k} = -\frac{\omega^2 cL}{4} = -\left(\frac{\omega}{\omega_0}\right)^2 = -K_0^2 \qquad \text{Here, } \omega_0 = \frac{2}{\sqrt{LC}}, \quad K_0 = \frac{\omega}{\omega_0}$$

P. B.

$$0 \leq K_0 \leq 1 \qquad (1 \geq k \geq -1) \qquad \therefore\; 0 < \omega \leq \omega_0 \;\text{ or }\; 0 < \omega \leq \frac{2}{\sqrt{LC}}$$

$$\text{Here,} \quad \sinh\frac{\theta}{2} = \sinh\frac{\beta}{2}\cos\frac{\alpha}{2} + j\cosh\frac{\beta}{2}\sin\frac{\alpha}{2} \qquad (\beta = 0)$$

$$\therefore\; \sin\frac{\alpha}{2} = \frac{\omega}{\omega_0} = K_0$$

A.B.

(A.B−Ⅰ) $K_0 \geq 1$ $(k \leq -1)$ $\therefore$ $\omega > \omega_0$ $\alpha = \frac{2n\pi}{2}$

$$\therefore \quad \cosh\frac{\beta}{2} = \frac{\omega}{\omega_0} = K_0$$

So the attenuation of the circuit becomes large in proportion to the magnitude of K_0.

(A.B.−Ⅱ) $K_0 \leq 0$ $(k = 1 - \coth^2\beta \leq 0)$

when $k=0$, β is equal to infinite and $\alpha = \frac{(2n+1\pi)}{2}$

(iii) Inverse calculation

This filter's C. F. (ω_0) and attenuation constant are determined by L and C with the aid of the coupling parameter, as mentioned above. when $\omega=0$ $(K=0)$, the image impedance of the circuit is as follows.

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \therefore \quad Z_{0(a)} = \sqrt{\frac{Z}{Y}}\sqrt{1+K}, \qquad Z_{0(b)} = \sqrt{\frac{Z}{Y}} \cdot \frac{1}{\sqrt{1+K}}$$

where $\omega_0 = \frac{2}{\sqrt{LC}}$, so the desired element's constants are given as follows:

$$L = \frac{2}{\omega_0} Z_0, \qquad , \qquad C = \frac{2}{\omega_0} \cdot \frac{1}{Z_0}$$

The utility of the generalized coupling coefficient (k) in the filter network analysis is as follows.

(a) If we find k of any filter network, the filter characteristics of this circuit can be analysed entirely on the base of this k, and from the relation of θ and k, the physical meanings of this filter network become very easy to understand.

(b) In any filter network analysis, the conception about P.B., A.B. and C.F. of the filter network based upon the normalized frequency and the image voltage transfer constant, coincides with the idea, which contains the coupling coefficient (k), that we consider any filter nework as s coupling 4-terminal network.

§5. Conclusion

If we consider any 4-terminal network as a coupling network, the parameter which contains k is very useful in the following calculations, as mentioned above, that is, (a) determination of 4-terminal constants about any complex network, (b) circuit calculations (e.g. calculation of balanced condition etc.) (c) analysis of oscillatory circuit and filter circuit. On the frequency characteristics of k and the application to the circuit design, we wish to continue our study in future.

I take this opportunity of many thanks to Prof. S. Otaka, S. Okada and T. Hasegawa and others for very helpful suggestions in regard to this problem.

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Foot-notes.

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一般化した結合係数kに就て

鈴木道也

内容梗概

一般に4端子回路網を結合回路網の一環として取扱い，回路の結合度を表現する結合係数kを一般化して考察すると回路計算及解析上種々便で且その物理的意味の把握が容易である。本稿ではかかる観点に基いて変成器理論における該係数の占値領域を拡張し,2,3の応用を示した。

	7	$-e_\lambda$	$-e'_\lambda$
	9	$e_\lambda - r_{\lambda\kappa} e^\kappa$	$e''_\lambda - r_{\lambda\kappa} e''^\kappa$
	10	oquivalent	equivalent
	—5	E_λ	E'_λ
	—1	$E_\lambda = r_{\lambda\kappa} E^\kappa$	$E''_\lambda = r_{\lambda\kappa} E''^\kappa$
	middle 8	grot	g rot
	right 6	$-\mathbf{e}_a = -\mathrm{grad} V$	$-\mathbf{e}_a = -\mathrm{grad}\ V_e$

「摘 要」 全文

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459	—10	points 1-1′2′-2′	points 1-1′,2-2′
460	12	Y-matrix representation	2 Y-matrix representation
	—4	in thc following way,	in the following way,
464 (462)	—22	terminal vol tages	terminal voltages
	—18	Opem circnit	Open circuit
465 (463)	6	巴in -tyee	in 卌-type
	13	networt	network
	19	indicatedi	indicated
	—13	int he	in the
	—4	Under tde	under the
462 table (464) ～ 465		k	k
		K-matrix's elements of Type 〒 $A_2 = 1 + \frac{Z_b Z_c}{}$	〃 $A_2 = 1 + \frac{Z_b Z_c}{\Box}$
		K-matrix's elements of Type ℿ $B = \frac{Z_b Z_a + Z_c) + Z_a Z_c}{Z^a}$	〃 $B = \frac{Z_b(Z_a + Z_c) + Z_a Z_c}{Z_a}$
		K-matrix's elements of Type X $A_2 = \frac{(Z_a + Z_b)(Z\ Z_d)}{Z_b Z_c\ Z_a Z}$	〃 $A_2 = \frac{(Z_a + Z_b)(Z_c + Z_d)}{Z_b Z_c - Z_a Z_d}$
		Zm of Type 卌 $\frac{Z_a Z_c Z_d}{(Z_a + Z_b)(Z_c + Z_d) + Z_c Z_a}$	〃 $\frac{Z_a Z_c Z_d}{(Z_a + Z_b)(Z_c + Z_d) + Z_c Z_d}$
466	—16	4-terminal -type	4-terminal ℿ-type
	—4	conditio	condition
468	—12	,then $\cos\alpha = \pm 1, k = -1$ and⋯	,then $\cos\alpha = \pm 1,\ k = \pm\ 1$ and⋯⋯
	—7	resistve	resistive
	—4	$\frac{1}{k^2} \geq 1$ and $\frac{1}{k^2} \leq 0$	$\frac{1}{k^2} \geq 1$ and $\frac{1}{k^2} \leq 0$ (4.4)

469	6	$\tanh\theta=\tanh B<1$	$\tanh\theta=\tanh\beta<1$
469	14	vale	value
470	4	$1\leq\frac{1}{k^2}\leq 0\ (0\leq K\leq -1)$	$\frac{1}{k^2}\geq 1,\ \frac{1}{k^2}\leq 0 (K\geq 0,\ K\leq -1)$
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	—1,2,6	Sin, Sinh	sin, sinh
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	7	$\mathrm{k}=0$	$k=0$
	〃	$\alpha=\frac{(2n+1\pi)}{2}$	$\alpha=\frac{(2n+1)\pi}{2}$
	—17	thc relation	the relation
	—12	as s coupling	as a coupling