BRST Symmetric Gaugeon Formalism for Yang-Mills Fields[*]

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Abstract

We introduce BRST symmetries into Yokoyama's gaugeon formalisms for Yang-Mills gauge fields. Owing to the BRST symmetry, Yokoyama's physical subsidiary conditions are replaced by a single condition of the Kugo-Ojima type. As a result, the formalisms become applicable even in the background gravitational field.

1 Introduction

In the standard formalism of canonically quantized gauge theories [1, 2] we do not consider the gauge transformation which connects field operators of different gauges. There are no such gauge freedom in the quantum theory since the quantum theory is defined only after the gauge fixing. In other words, the Fock space defined in a particular gauge is not wide enough to realize the quantum gauge freedom.

Yokoyama's gaugeon formalism [3]-[9] provides a wider framework in which we can consider the quantum gauge transformation among a family of Lorentz covariant linear gauges. In this formalism a set of extra fields, so called gaugeon fields, is introduced as the quantum gauge freedom. This theory was first proposed for the quantum electrodynamics [3, 4, 5] to resolve the problem of gauge parameter renormalization [10]. It was also applied later to the Yang-Mills theory [6, 9]. Owing to the quantum gauge freedom it becomes very easy to check the gauge parameter independence of the physical S-matrix [7]. The

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gauge dependence of the wave-function renormalization constant was also investigated in this formalism [8].

We should ensure that the gaugeon modes do not contribute to the physical processes. In fact, the gaugeon fields yield negative normed states that would lead to the negative probability [3]. To remove these unphysical gaugeon modes Yokoyama imposed a Gupta-Bleuler type subsidiary condition [3, 6, 9]. However, this type of condition is not applicable if interaction exists for the gaugeon fields. Especially, we cannot use the condition in the background gravitational field.

Yokoyama's subsidiary condition can be improved if we can introduce the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [11] for the gaugeon fields. Izawa has proposed a BRST symmetric Lagrangian for the gaugeon formalism in the quantum electrodynamics (QED) [12]. Independently of Izawa's work, we also have presented a BRST symmetric gaugeon formalism for the QED [13]. Both theories[1] include Faddeev-Popov (FP) ghosts for the gaugeon fields as well as the usual FP ghosts. As a result, the theories have larger BRST symmetry and corresponding conserved charges (BRST charges). Using the BRST charges, we can replace the Yokoyama's subsidiary condition by a single Kugo-Ojima type condition [2], which is applicable even to the interacting case.

In the present paper, we extend our BRST symmetric gaugeon formalism for QED to the Yang-Mills gauge theories. We do this by simply introducing BRST symmetry into the original gaugeon formalisms for the Yang-Mills fields. There are two types of gaugeon formalisms for Yang-Mills fields so far. One of them was proposed by Yokoyama [6]. It has a group vector valued gauge fixing parameter $\alpha = (\alpha^a)$. The gauge fixing is different from the standard one in the sense that it breaks not only the local gauge symmetry but also the rigid gauge symmetry. The other type of the formalism was proposed by Yokoyama, Takeda and Monda [9]. It has a (group scalar valued) single gauge fixing parameter α. Thus the gauge fixing does not violate the rigid gauge symmetry; though the Lagrangian has nonpolynomial interaction terms. In the present paper we introduce larger BRST symmetry into both types of the gaugeon formalism for the Yang-Mills fields.

The notation and convention used in this paper are the following. The metric we use is $g_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$. The gauge group we consider is a n-dimensional compact Lie group, the generators of which are denoted by T^a ($a = 1, 2, \ldots n$). Latin letters $a, b, c, \ldots$ denote the group vector indices, while Greek letters $\mu, \nu, \lambda, \ldots$ express the space-time indices which run from 0 to 3. The summation convention is assumed for both group vector indices and space-time indices. The generators satisfy

$$(T^a)^\dagger = T^a, \qquad [T^a, T^b] = if^{abc}T^c.$$

Here the structure constant f^{abc} is totally antisymmetric since we assume the normalization for the generators as

$$\text{tr } T^aT^b = \frac{1}{2}\delta^{ab}.$$

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[1]For the relation between Izawa's theory and ours, see Refs.[13, 14].

2 Gaugeon formalism with a group vector valued gauge parameter

In the formalism we discuss in this section, the group vector valued gauge fixing parameter $\alpha = (\alpha^a)$ is introduced.[2] As a result, Yokoyama's gaugeon fields Y and Y_* are group scalar, while the Nakanishi–Lautrup (Lagrange multiplier) field $\boldsymbol{B} = (B^a)$ and FP-ghost fields $\boldsymbol{c} = (c^a)$ and $\boldsymbol{c}_* = (c_*^a)$ are group vector valued.

2.1 Yokoyama's theory

Yokoyama's Lagrangian for the Yang-Mills field $\boldsymbol{A}_\mu = (A_\mu^a)$ is given by

$$
\begin{aligned}
L_{\mathrm{Y}} &= -\frac{1}{4} \boldsymbol{F}^{\mu\nu} \boldsymbol{F}_{\mu\nu} - \boldsymbol{A}^\mu \nabla_\mu \boldsymbol{B} + \partial^\mu Y_* \partial_\mu Y + \frac{\varepsilon}{2}(Y_* + \boldsymbol{\alpha} \boldsymbol{B})^2 \\
&\quad -i\nabla^\mu \boldsymbol{c}_* D_\mu \boldsymbol{c} + L_{\mathrm{matter}}(\psi, D_\mu \psi),
\end{aligned}
\tag{2.1}
$$

$$
\boldsymbol{F}_{\mu\nu} = \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + g\boldsymbol{A}_\mu \times \boldsymbol{A}_\nu,
\tag{2.2}
$$

$$
D_\mu \boldsymbol{c} = \partial_\mu \boldsymbol{c} + g\boldsymbol{A}_\mu \times \boldsymbol{c}, \qquad D_\mu \psi = (\partial_\mu - ig A_\mu^a T^a)\psi,
\tag{2.3}
$$

$$
\nabla_\mu \boldsymbol{V} = \partial_\mu \boldsymbol{V} + g\boldsymbol{\alpha}\partial_\mu Y \times \boldsymbol{V}, \qquad (\boldsymbol{V} = \boldsymbol{B}, \ \boldsymbol{c}_*)
\tag{2.4}
$$

where $\boldsymbol{\alpha}$ is the group vector valued gauge fixing parameter, g the coupling constant, ε a sign factor $(= \pm 1)$, $L_{\mathrm{matter}}(\psi, D_\mu \psi)$ the Lagrangian of a matter field ψ minimally coupled with $\boldsymbol{A}_\mu$, $\boldsymbol{F}_{\mu\nu}$ the field strength, Y and Y_* the gaugeon field and its associated field subject to the Bose-Einstein statistics, $\boldsymbol{c}$ and $\boldsymbol{c}_*$ are the FP-ghost fields subject to the Fermi-Dirac statistics, D_μ is the covariant derivative, and ∇_μ is called the *form covariant derivative*. Since the gauge parameter $\boldsymbol{\alpha}$ is group vector valued, the gauge field propagator is different from the standard one. In fact, the tree level propagator in the momentum space is given by

$$
\langle A_\mu^a A_\nu^b \rangle \sim \frac{\delta^{ab}}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \varepsilon \alpha^a \alpha^b \frac{k_\mu k_\nu}{(k^2)^2},
\tag{2.5}
$$

which does not coincide with the propagator of the standard formalism unless the Landau gauge $(\boldsymbol{\alpha} = 0)$ is chosen.

The Lagrangian (2.1) admits q-number gauge transformations. Under the infinitesimal field transformation

$$
\begin{aligned}
\hat{A}_\mu &= \boldsymbol{A}_\mu + \tau D_\mu(\boldsymbol{\alpha} Y) = \boldsymbol{A}_\mu + \tau(\boldsymbol{\alpha}\partial_\mu Y + g\boldsymbol{A}_\mu \times \boldsymbol{\alpha} Y), \\
\hat{\psi} &= (1 - ig\tau\alpha^a Y T^a)\psi,
\end{aligned}
$$

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[2]We use the group vector notation in this section: Letters in boldface denote group vectors. For any two group vectors $\boldsymbol{V} = (V^a)$ and $\boldsymbol{W} = (W^a)$, we have an inner product

$$
\boldsymbol{V}\boldsymbol{W} = V^a W^a,
$$

and an exterior product

$$
\boldsymbol{V} \times \boldsymbol{W} = f^{abc} V^b W^c.
$$

$$\hat{B} = B + \tau g B \times \alpha Y, \tag{2.6}$$
$$\hat{Y} = Y, \qquad \hat{Y}_* = Y_* - \tau \alpha B,$$
$$\hat{c} = c + \tau g c \times \alpha Y, \qquad \hat{c}_* = c_* + \tau g c_* \times \alpha Y$$

with τ being an infinitesimal parameter (group scalar), the Lagrangian is *form invariant*, that is, it transforms as

$$L_{\rm Y}(\phi^A; \alpha) = L_{\rm Y}(\hat{\phi}^A; \hat{\alpha}), \tag{2.7}$$

where ϕ^A stands for any of the fields we are considering and $\hat{\alpha}$ is defined by

$$\hat{\alpha} = (1 + \tau)\alpha. \tag{2.8}$$

Similarly, under the infinitesimal group vector rotation

$$\hat{A}_\mu = A_\mu + A_\mu \times \omega, \qquad \hat{\psi} = (1 - ig\omega^a T^a)\psi,$$
$$\hat{B} = B + B \times \omega,$$
$$\hat{Y} = Y, \qquad \hat{Y}_* = Y_*, \tag{2.9}$$
$$\hat{c} = c + c \times \omega, \qquad \hat{c}_* = c_* + c_* \times \omega$$

with $\omega = (\omega^a)$ being an infinitesimal group vector parameter, the Lagrangian transforms as (2.7) with $\hat{\alpha}$ given by

$$\hat{\alpha} = \alpha + \alpha \times \omega. \tag{2.10}$$

The form invariance (2.7) (under (2.6) and (2.9)) means that ϕ^A and $\hat{\phi}^A$ satisfy the same field equation except for the parameter α which should be replaced by $\hat{\alpha}$ for the $\hat{\phi}^A$ field equation. Thus, we can shift and rotate the gauge parameter α by the q-number gauge transformations (2.6) and (2.9). Note that the sign factor ε cannot be changed by these transformations.

The Lagrangian (2.1) is invariant under the following BRST transformation:

$$\delta_{\rm B} A_\mu = D_\mu c, \qquad \delta_{\rm B}\psi = -igc^a T^a \psi,$$
$$\delta_{\rm B} c = -\frac{g}{2}c \times c,$$
$$\delta_{\rm B} c_* = iB, \qquad \delta_{\rm B} B = 0, \tag{2.11}$$
$$\delta_{\rm B} Y = \delta_{\rm B} Y_* = 0,$$

which obviously satisfies the nilpotency, $\delta_{\rm B}^2 = 0$. Corresponding to this invariance, there exists a Noether current $J_{\rm B}^\mu$ satisfying the conservation law

$$\partial_\mu J_{\rm B}^\mu = 0. \tag{2.12}$$

Thus we can define the BRST charge by

$$Q_{\rm B} = \int d^3x J_{\rm B}^0, \tag{2.13}$$

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which satisfies the nilpotency $Q_B{}^2 = 0$.

To remove the unphysical modes and define physical states, Yokoyama imposed two kinds of subsidiary conditions:

$$Q_B \, |\text{phys}\rangle = 0, \tag{2.14}$$

$$(Y_* + \alpha B)^{(+)} \, |\text{phys}\rangle = 0. \tag{2.15}$$

As shown by Kugo and Ojima [2], the first condition removes the unphysical gauge field modes from the total Fock space. The nilpotency and conserving property of Q_B is essential in proving that this condition works well. The second condition is a Gupta-Bleuler type condition [1]; the superscript $(+)$ denotes the positive frequency part. It removes the unphysical gaugeon modes. In this context, it is important that the combination

$$\Lambda = Y_* + \alpha B \tag{2.16}$$

satisfies the free field equation

$$\Box \Lambda = 0. \tag{2.17}$$

Owing to the free equation, the decomposition of Λ into the positive and negative frequency parts is well-defined. However, once we consider the gravitational interaction, the decomposition of Λ into $\Lambda^{(\pm)}$ is no longer well-defined. This is the limitation of the Gupta-Bleuler type subsidiary condition; the Kugo-Ojima type condition based on the conserved charge has no limitation of this kind.

2.2   BRST symmetric theory

As a BRST symmetric version of (2.1) we propose the following Lagrangian:

$$
\begin{aligned}
L = \ & -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - A^\mu \nabla_\mu B + \partial^\mu Y_* \partial_\mu Y + \frac{\varepsilon}{2}(Y_* + \alpha B)^2 \\
& -i\nabla^\mu c_* D_\mu c - i\partial^\mu K_* \partial_\mu K + L_{\text{matter}}(\psi, D_\mu \psi),
\end{aligned} \tag{2.18}
$$

where group scalars K and K_*, subject to the Fermi-Dirac statistics, are FP-ghost fields for the gaugeon fields Y and Y_*.

By introducing K and K_* we are able to extend the BRST transformation so that the gaugeon fields are also transformed. We consider the following larger BRST transformation:

$$
\begin{aligned}
\delta_B A_\mu &= D_\mu c, & \delta_B \psi &= -igc^a T^a \psi, \\
\delta_B c &= -\frac{g}{2} c \times c, & & \\
\delta_B c_* &= iB, & \delta_B B &= 0, \\
\delta_B Y &= K, & \delta_B K &= 0, \\
\delta_B K_* &= -iY_*, & \delta_B Y_* &= 0,
\end{aligned} \tag{2.19}
$$

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which satisfies $\delta_B{}^2 = 0$. Because of the nilpotency, the invariance under this transformation can be easily seen if we rewrite the Lagrangian as

$$
\begin{aligned}
L \;=\; & -\frac{1}{4}\boldsymbol{F}^{\mu\nu}\boldsymbol{F}_{\mu\nu} + L_{\mathrm{matter}}(\psi, D_\mu\psi) \\
& -i\delta_B\left[\boldsymbol{c}_*\left(\nabla^\mu\boldsymbol{A}_\mu - \frac{\varepsilon\alpha}{2}(Y_* + \alpha B)\right) + K_*\left(\Box Y - \frac{\varepsilon}{2}(Y_* + \alpha B)\right)\right]. \quad (2.20)
\end{aligned}
$$

The BRST current is now given by

$$
J_B^\mu = -\boldsymbol{F}^{\mu\nu}D_\nu\boldsymbol{c} - i\,\frac{g}{2}\,\nabla^\mu\boldsymbol{c}_*(\boldsymbol{c}\times\boldsymbol{c}) - (D^\mu\boldsymbol{c})\boldsymbol{B} - Y_*D^\mu K, \quad (2.21)
$$

which yields the conserved BRST charge $Q_B = \int d^3x J_B^0$.

As for the q-number gauge transformation, we now consider the following field transformation:

$$
\begin{aligned}
\hat{\boldsymbol{A}}_\mu &= \boldsymbol{A}_\mu + \tau D_\mu(\alpha Y) & \hat{\psi} &= (1 - ig\tau\alpha^a T^a Y)\psi, \\
\hat{\boldsymbol{B}} &= \boldsymbol{B} + \tau g\boldsymbol{B}\times\alpha Y - i\tau g\boldsymbol{c}_*\times\alpha K, \\
\hat{Y} &= Y, & \hat{Y}_* &= Y_* - \tau\alpha B, \\
\hat{\boldsymbol{c}} &= \boldsymbol{c} + \tau g\boldsymbol{c}\times\alpha Y + \tau\alpha K, & \hat{\boldsymbol{c}}_* &= \boldsymbol{c}_* + \tau g\boldsymbol{c}_*\times\alpha Y \\
\hat{K} &= K, & \hat{K}_* &= K_* - \tau\alpha c.
\end{aligned}
\quad (2.22)
$$

Under this transformation [and the rotation (2.9)] the Lagrangian (2.18) is again form invariant:

$$
L(\phi^A;\boldsymbol{\alpha}) = L(\hat{\phi}^A;\hat{\boldsymbol{\alpha}}) \quad (2.23)
$$

with $\hat{\boldsymbol{\alpha}}$ given by (2.8) [or by (2.10)]. Thus the theories with different gauge fixing parameters $\boldsymbol{\alpha}$ are included in one theory described by the Lagrangian (2.18).

The physical subsidiary condition becomes now simpler. We impose a single condition,

$$
Q_B |\mathrm{phys}\rangle = 0. \quad (2.24)
$$

Since our BRST operator acts on the gaugeon fields as well as usual gauge fields, the condition removes all the unphysical modes. (For example, as seen from (2.19), Y, Y_*, K and K_* form a BRST quartet [2], which is known to appear only as zero-normed states in the physical subspace.) Thus we are able to avoid the Gupta-Bleuler type subsidiary condition. Consequently, our physical condition works well even in the background gravitational filed.

3　Gaugeon formalism with a single gauge parameter

In the present section we consider the gaugeon formalism in which the gauge fixing parameter is a group scalar. In this sense the theory is more similar with the standard formalism [2] than the theory discussed in the last section. And gaugeon fields have also

group vector indices (Y^a and Y_*^a). We use the matrix notation for the group vector in this section. For any group vector $\boldsymbol{V} = (V^a)$, we define[3]

$$V = V^a T^a.$$

3.1 Yokoyama-Takeda-Monda theory

The Lagrangian of Yokoyama-Takeda-Monda theory [9] is given by

$$
\begin{aligned}
L_{\text{YTM}} &= 2\text{tr}\left\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (A^\mu - F^\mu)\nabla_\mu B\right\} \\
&+ 2\text{tr}\left\{\partial^\mu Y_* \partial_\mu Y + \frac{\varepsilon}{2}Y_*^2 - i\nabla^\mu c_* D_\mu c\right\} + L_{\text{matter}}(\psi, D_\mu \psi),
\end{aligned}
\tag{3.1}
$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{3.2}$$

$$D_\mu V = \partial_\mu V - ig[A_\mu, V], \tag{3.3}$$

$$\nabla_\mu V = \partial_\mu V - ig\alpha[F_\mu, V]. \tag{3.4}$$

$$-ig\alpha F_\mu = S^{-1}(\alpha)\, \partial_\mu S(\alpha), \tag{3.5}$$

$$S(\alpha) = \exp(-ig\alpha Y), \tag{3.6}$$

where α is the group scalar valued gauge fixing parameter, ε again a sign factor, and $Y = Y^a T^a$ and $Y_* = Y_*^a T^a$ are the Lie algebra valued gaugeon fields. The tree level propagator of gauge fields is given by

$$\langle A_\mu^a A_\nu^b\rangle \sim \frac{\delta^{ab}}{k^2}\left(g_{\mu\nu} + (\varepsilon\alpha^2 - 1)\frac{k_\mu k_\nu}{k^2}\right), \tag{3.7}$$

which coincides with the propagator of the standard formalism though the nonperturbative propagator differs from the standard one. Note that $S(\alpha)$ has its value in the group and consequently $F_\mu(= F_\mu^a T^a)$ lives in the Lie algebra. As seen from (3.5), F_μ is a non-polynomial function of Y. The renormalizability of this theory with such nonpolynomial interactions is discussed in Ref.[9].

The Lagrangian admits the q-number gauge transformation defined by

$$
\begin{aligned}
\hat{A}_\mu &= S^{-1}(\tau)\, A_\mu S(\tau) + \frac{i}{g}S^{-1}(\tau)\, \partial_\mu S(\tau), \\
\hat{\psi} &= S^{-1}(\tau)\, \psi, \\
\hat{V} &= S^{-1}(\tau)\, V S(\tau), \qquad (V = B, c, c_*) \\
\hat{W} &= W, \qquad\qquad (W = Y, Y_*)
\end{aligned}
\tag{3.8}
$$

with τ being a finite parameter. Under this field transformation, the Lagrangian is form invariant:

$$L_{\text{YTM}}(\phi^A; \alpha) = L_{\text{YTM}}(\hat{\phi}^A; \hat{\alpha}), \tag{3.9}$$

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[3]In this notation, a commutator corresponds to the exterior product: $-i[V, W] = (\boldsymbol{V} \times \boldsymbol{W})^a T^a$.

where ϕ^A stands for any of the fields and $\hat{\alpha}$ is defined by

$$\hat{\alpha} = \alpha + \tau. \tag{3.10}$$

The Lagrangian (3.1) is invariant under the following BRST transformation:

$$
\begin{aligned}
\delta_{\mathrm{B}} A_\mu &= D_\mu c, & \delta_{\mathrm{B}} \psi &= -igc\psi, \\
\delta_{\mathrm{B}} B &= \delta_{\mathrm{B}} Y = \delta_{\mathrm{B}} Y_* = 0, \\
\delta_{\mathrm{B}} c &= igc^2, & \delta_{\mathrm{B}} c_* &= iB,
\end{aligned}
\tag{3.11}
$$

which obviously satisfies the nilpotency, $\delta_{\mathrm{B}}{}^2 = 0$. Corresponding to this symmetry, we have a conserved BRST charge Q_{B}.

To remove the unphysical modes and define physical states, Yokoyama, Takeda and Monda imposed the following conditions:

$$Q_{\mathrm{B}} |\mathrm{phys}\rangle = 0, \tag{3.12}$$

$$Y_*^{(+)} |\mathrm{phys}\rangle = 0. \tag{3.13}$$

The first condition removes the unphysical modes of gauge field while the second eliminates unphysical gaugeon modes. It is essential in the second condition that the field Y_* satisfies the free field equation,

$$\Box Y_* = 0. \tag{3.14}$$

Owing to the free equation, the decomposition of Y_* into the positive and negative frequency parts $Y_*^{(\pm)}$ is well-defined. However, once we consider the gravitational interaction, the Gupta-Bleuler type condition (3.13) no longer works well.

3.2 BRST symmetric theory

As a BRST symmetric version of (3.1) we present a Lagrangian given by

$$
\begin{aligned}
L &= 2\mathrm{tr} \left\{ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (A^\mu - F^\mu) \nabla_\mu B + \partial^\mu Y_* \partial_\mu Y + \frac{\varepsilon}{2} Y_*{}^2 \right\} \\
&\quad + 2\mathrm{tr} \left\{ -i\nabla^\mu c_* D_\mu c - i\partial^\mu K_* \partial_\mu K \right\} + L_{\mathrm{matter}}(\psi, D_\mu \psi),
\end{aligned}
\tag{3.15}
$$

where $K = K^a T^a$ and $K_* = K_*^a T^a$ have been introduced as Lie algebra valued FP-ghost fields for the gaugeon fields Y and Y_*.

We may consider the q-number transformation defined by

$$
\begin{aligned}
\hat{A}_\mu &= S^{-1}(\tau) A_\mu S(\tau) + \frac{i}{g} S^{-1}(\tau)\, \partial_\mu S(\tau), \\
\hat{\psi} &= S^{-1}(\tau)\, \psi, \\
\hat{V} &= S^{-1}(\tau)\, V S(\tau), \\
\hat{W} &= W,
\end{aligned}
\tag{3.16}
$$

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where, and in the following, V stands for B, c, and c_* and W denotes Y, Y_*, K, and K_*. The Lagrangian is form invariant under this transformation:

$$L(\hat{\phi}^A; \hat{\alpha}) = L(\phi^A; \alpha) \tag{3.17}$$

with $\hat{\alpha}$ being $\hat{\alpha} = \alpha + \tau$. To check the form invariance (3.17) we have used the identities,

$$\begin{aligned}
\hat{A}_\mu - \hat{\alpha}\hat{F}_\mu &= S^{-1}(\tau)\,(A_\mu - \alpha F_\mu)S(\tau), \\
\hat{F}_{\mu\nu} &= S^{-1}(\tau)\,F_{\mu\nu}S(\tau), \\
\widehat{\nabla_\mu V} &= S^{-1}(\tau)\,\nabla_\mu V S(\tau), \\
\widehat{D_\mu V} &= S^{-1}(\tau)\,D_\mu V S(\tau).
\end{aligned} \tag{3.18}$$

The BRST transformation we propose here is

$$\begin{aligned}
\delta_{\mathrm{B}} A_\mu &= D_\mu(c + \alpha\mathcal{K}), \\
\delta_{\mathrm{B}} \psi &= -ig(c + \alpha\mathcal{K})\psi, \\
\delta_{\mathrm{B}} B &= ig\alpha[\mathcal{K}, B], \\
\delta_{\mathrm{B}} c &= ig\left\{\frac{1}{2}c + \alpha\mathcal{K}, c\right\}, \\
\delta_{\mathrm{B}} c_* &= -iB + ig\alpha\{\mathcal{K}, c_*\}, \\
\delta_{\mathrm{B}} Y &= K, \\
\delta_{\mathrm{B}} Y_* &= 0, \\
\delta_{\mathrm{B}} K &= 0, \\
\delta_{\mathrm{B}} K_* &= -iY_*,
\end{aligned} \tag{3.19}$$

where $\mathcal{K}$ is defined by

$$\mathcal{K} = K^a F^a, \tag{3.20}$$

$$-ig\alpha F^a = S^{-1}(\alpha)\,\frac{\partial}{\partial Y^a}S(\alpha). \tag{3.21}$$

By using the identities

$$\delta_{\mathrm{B}} F_\mu = \nabla_\mu \mathcal{K}, \tag{3.22}$$

$$\delta_{\mathrm{B}} \mathcal{K} = ig\alpha\mathcal{K}^2, \tag{3.23}$$

we can easily check the nilpotency of our BRST transformation (3.19). Furthermore we can show the BRST invariance of the Lagrangian since we may rewrite the Lagrangian as

$$\begin{aligned}
L &= 2\mathrm{tr}\left\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right\} + L_{\mathrm{matter}}(\psi, D_\mu\psi) \\
&\quad -i\delta_{\mathrm{B}}\left[2\mathrm{tr}\left\{c_*\partial^\mu(A_\mu - \alpha F_\mu) - \partial^\mu K_*\partial_\mu Y - \frac{\varepsilon}{2}K_*Y_*\right\}\right].
\end{aligned} \tag{3.24}$$

We have thus a conserved and nilpotent BRST charge Q_{B} corresponding to the symmetry under (3.19). Using the BRST charge we impose the physical subsidiary condition as

$$Q_{\mathrm{B}}\,|\mathrm{phys}\rangle = 0, \tag{3.25}$$

by which we replace the two subsidiary conditions (3.12) and (3.13) of Yokoyama, Takeda and Monda. In particular, we do not need any Gupta-Bleuler type subsidiary condition. Consequently, our theory is applicable even in the background gravitational field.

4   Summary and remarks

We have presented two kinds of gaugeon formalisms for Yang-Mills fields with larger BRST symmetries. One is an extension of Yokoyama's theory [6] in which a group vector valued parameter is used in the gauge fixing term. The other is an extension of the theory by Yokoyama, Takeda and Monda [9] which has a group scalar valued gauge fixing parameter. By using BRST charges corresponding to the larger BRST symmetries, we have been able to replace the Yokoyama's physical subsidiary conditions by a single Kugo-Ojima type condition in each case. As a result, the formalism becomes applicable to the case of the background gravitational field.

We emphasize that in both cases (of sections 2 and 3) our physical condition is invariant under the q-number gauge transformation. As seen from (2.19) and (2.22), or from (3.19) and (3.16), the BRST transformation and the q-number gauge transformation commute with each other. This fact leads us to

$$\hat{Q}_{\mathrm{B}} = Q_{\mathrm{B}}, \tag{4.1}$$

that is, the BRST charge is invariant under the q-number gauge transformation. Consequently, our physical subsidiary conditions, and thus, our physical subspace are gauge invariant. In the case of quantum electrodynamics, this kind of structure of the physical subspace plays an essential role in the proof of the gauge parameter independence of the physical S-matrix [15].

Note added

After submitting this paper, we were informed of the work by M. Abe ("The Symmetries of the Gauge-Covariant Canonical Formalism of Non-Abelian Gauge Theories", Master Thesis, Kyoto University, 1985) in which he already proposed and studied the BRST-symmetrized Yokoyama-Takeda-Monda theory. His Lagrangian and BRST symmetry are the same as ours discussed in the section 3.

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